Actuarial Geometry: Volumetric and Temporal Diversification of Insurance Risk

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Outline

1. Insurance pricing frameworks
2. Insurance risk is not volumetrically diversifying
3. Insurance losses are not homogeneous with respect to volume
4. Homogeneous model is not even “locally” appropriate
5. Empirical data and supporting evidence
6. Four models based on Levy processes
7. Application to APAC region countries

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1. Insurance Pricing Frameworks

	Risk Theory	Finance	Actuarial
1900s	Bachelier		Bureau rates
1930s	Cramer-Lundberg Esscher		Bureau rates Bureau rates
1950s	Levy, Kolmogorov, Khintchine, Ito		…
1960s		Portfolio theory CAPM	Bureau rates Bailey investment inc.
1970s	Buhlmann Borch	Systemic vs. diversifiable risk	Ferrari, ROE 1978 US u/w profit
1980s	Geometric Brownian motion	Option pricing, no arbitrage, comparables	Catastrophe Models
1990s		Froot et al.	
2000s	Artzner et al. coherent measure of risk	Phillips, Cummins, Allen Myers-Read	Idiosyncratic risk matters (Froot 2001)
2010s	Wang transform Levy processes, optimal dividends	Zanjani	2004 US u/w profit Solvency II, ICA

2. Insurance Risk is not Volumetrically Diversifying

- Expected Loss ($) = Volume ($ / t) x Time (t)

- A(x,t) := random variable representing aggregate losses from volume x insured for time t
  - E[A(x,t)] = xt = expected loss

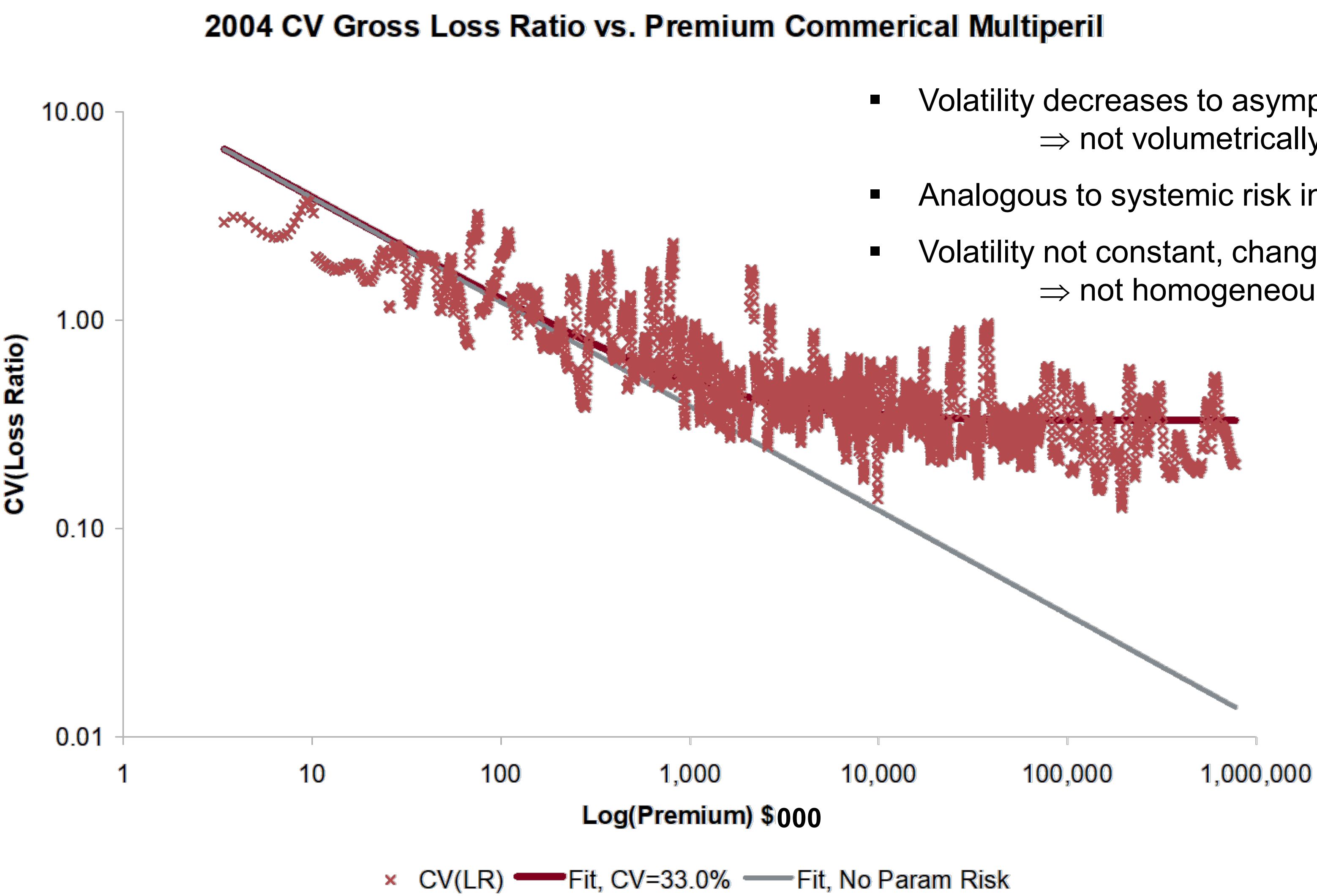
- Insurance risk is not volumetrically diversifying, meaning
  - CV(A(x,t)) does not tend to zero as x increases, for fixed t
  - Recall coefficient of variation = CV = standard deviation / mean

- Practical meaning
  - It is impossible to diversify away all insurance risk by growing larger

- How to investigate?
  - CV(A) = CV(A / p) = CV(loss ratio), p = fixed premium
  - Look at volatility in loss ratio with volume
  - Premium (and company) effects can be removed using an ANOVA; does not change conclusions

- Data source: Aon Benfield Insurance Risk Study global database of regulatory data from 49 countries
  - Represents over 90 percent of global P&C premium

2. Risk is not Volumetrically Diversifying



- Volatility decreases to asymptote > 0
  $\Rightarrow$ not volumetrically diversifying
- Analogous to systemic risk in stock portfolio
- Volatility not constant, changes shape
  $\Rightarrow$ not homogeneous

3. Insurance Losses are not Homogeneous with Respect to Volume

- **Homogeneous** model: $A(x,t) = xR_t$
  - R_t a "return" random variable independent of volume x
  - For assets x is position size and R_t is return or unit price
  - Introduces a natural vector space structure for assets, with basis the return vectors $R_{i,t}$

- Homogeneity implies
  - Shape of aggregate loss distribution independent of volume
  - No volume based diversification
  - A(x,t) has constant coefficient of variation (volatility) with x

- Homogeneous models are not appropriate for insurance
  - Consider probability of zero losses: $Pr(xX=0) = Pr(X=0)$
  - Implies the probability of observing a zero loss is **independent of volume x**

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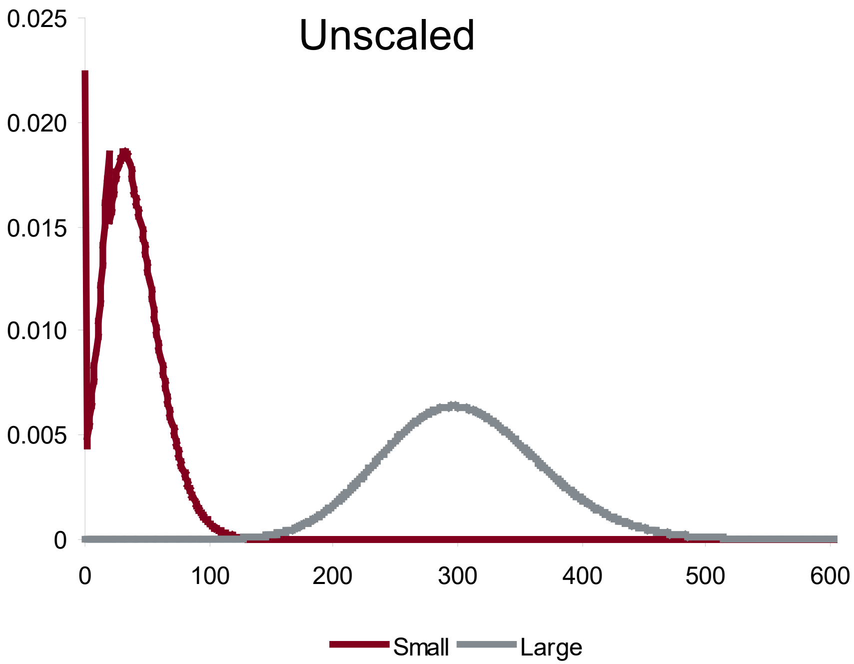
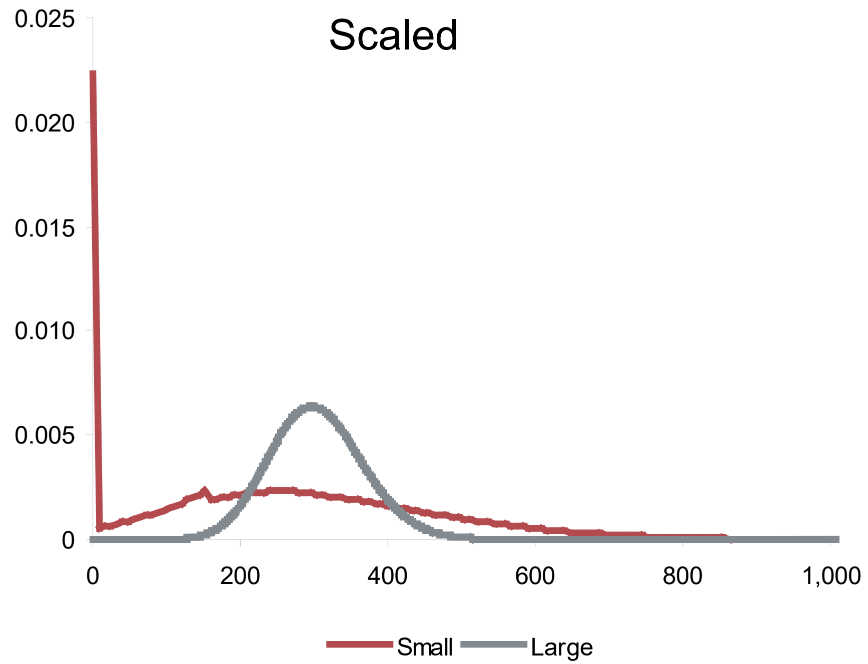
3. Insurance Losses are not Homogeneous with Respect to Volume





- Compound Poisson aggregate losses, average severity 10
  - Small: claim count 4
  - Large: claim count 32

- Homogeneous distributions would be indistinguishable in scaled plot
  - Note decrease in variance on right hand plot

- Matlab code: ifft(exp(4 * (fft(severity) – 1)))

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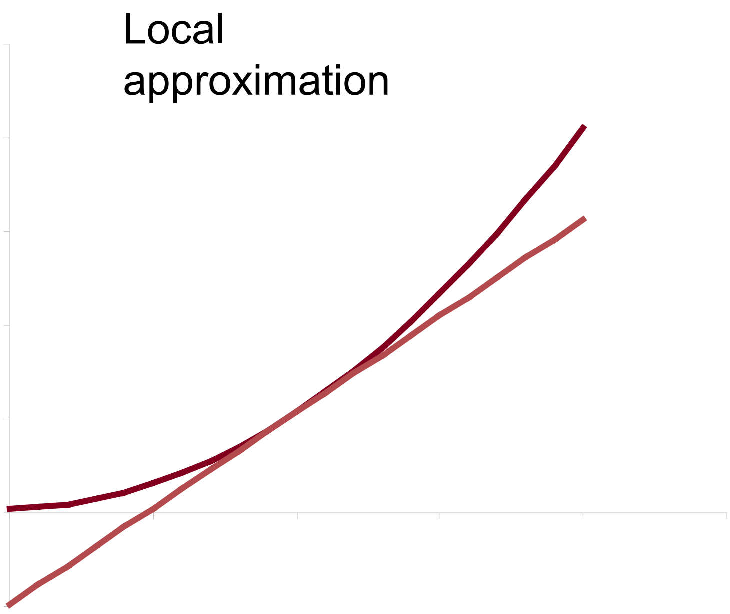
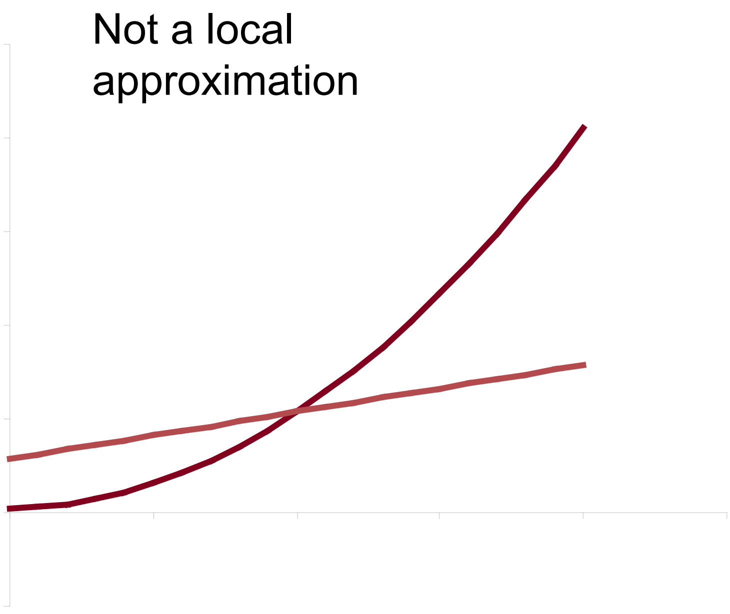
4. Homogeneity is not "Locally" Appropriate



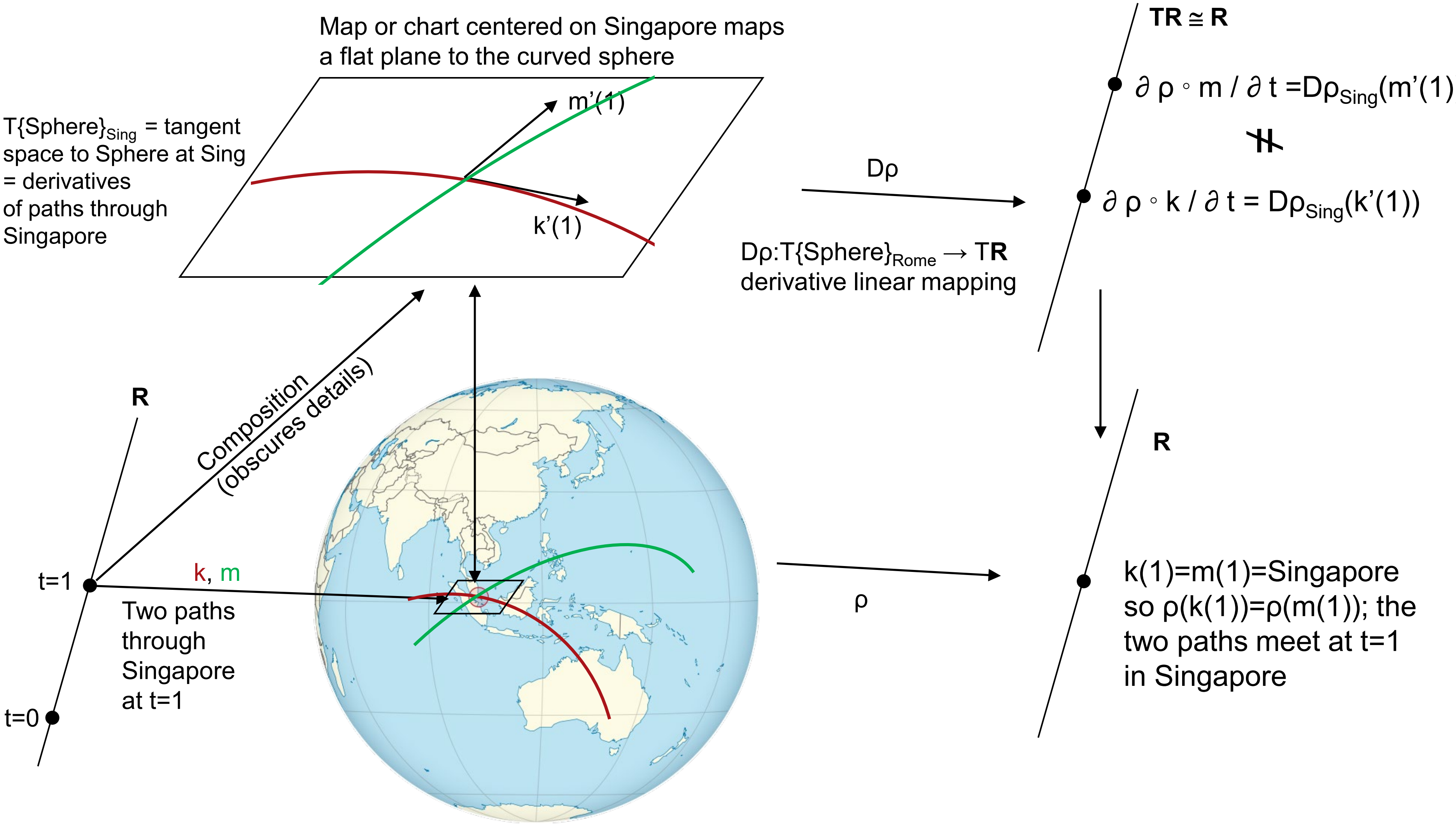


- Consider tX_1 as a homogeneous approximation to a process X_t, agreeing at t=1
- Local approximation: one holding to first order in a neighborhood of a point
  – First-order equality required by any theory considering derivatives or marginal impacts
  – Euler-theorem, gradient based methods of capital allocation
  – Equality at a point does not imply first order approximation
- Requires notion of **derivative** which requires a **direction**

4. Homogeneity is not "Locally" Appropriate: Example

- Have two maps from $[0,\infty) \rightarrow \{$ risks $\}$, agreeing at $t = 1$:
  - $m(t) = X(t)$, Poisson claim count t, (Glenn) Meyers embedding; not homogeneous
  - $k(t) = t\,X(1)$, asset or Kalkbrener embedding; homogeneous by construction
- Let $\rho : \{$ risks $\} \rightarrow \mathbf{R}$ be a real-valued risk measure
  - Standard deviation, downside risk, higher moment, percentile (=Value-at-Risk, VaR), TVaR
- Tasche, Denault, Fischer, Myers-Read,… show we should be interested in $\partial \rho / \partial t$, the rate of change of ρ with volume, in a given line of business or risk type
- Two compositions $\rho \circ k$, $\rho \circ m$: $[0,\infty) \rightarrow \{$ risks $\} \rightarrow \mathbf{R}$ both give single valued functions of a single real variable t, and we can often easily compute derivatives
- For ρ = standard deviation we have
  - $\rho \circ m(t) = \rho(m(t))$ = std dev(Poisson(t)) = $\sqrt{t}$, and $\boldsymbol{\partial (\rho \circ m) / \partial t = 1 / 2\sqrt{t}}$
  - $\rho \circ k(t) = \rho(k(t))$ = std dev(t Poisson(1)) = t, and so $\boldsymbol{\partial (\rho \circ k) / \partial t = 1}$

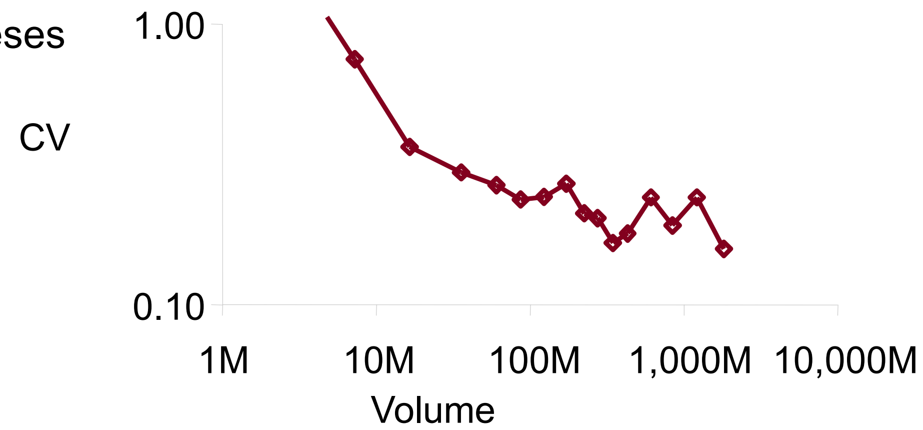
4. Homogeneity is not “Locally” Appropriate



Earth = { risks } = object
with complex geometry

5. Empirical Evidence

- We have seen the data supports two hypotheses
  - Risk is not homogeneous: i.e. CV not constant wrt volume
  - Risk is not volumetrically diversifying: CV has asymptote > 0



- Can we say more?

- Levy process based models, let X(.) be a Levy process

  - A(x,t) = X(xt) volumetric/temporal symmetry, diversifying
  - A(x,t) = X(xZ(t)) account for seasons, volumetric/temporal asymmetry
  - **A(x,t) = X(xCt)** E(C)=1, C is called a mixing variable, **symmetric, non-diversifying**
  - A(x,t) = X(xCZ(t)) combination, volumetric/temporal asymmetry

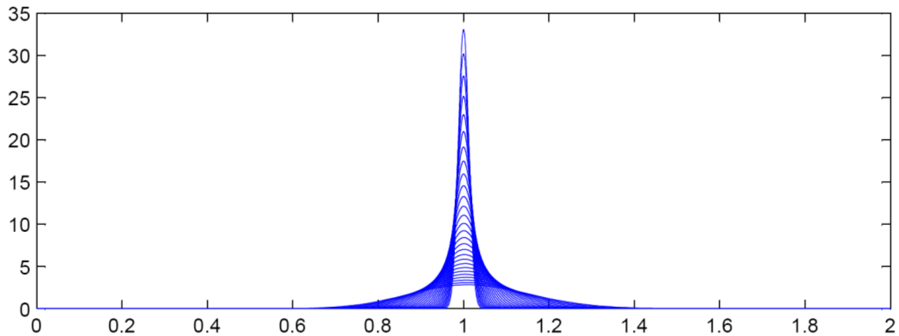
- The mixing variable appears unobservable, but can actually be derived from empirical data

- Tame severity distributions are irrelevant

Mixing Variables & the Distribution of Normalized Loss Ratios

- Mixed compound Poisson: $A = X_1+\ldots+X_N$, $N|C \sim \text{Poisson}(nC)$, $E(C)=1$
- Normalized Loss Ratio NLR = A / E(A) converges to distribution C
- Dichotomous behavior of normalized loss ratios

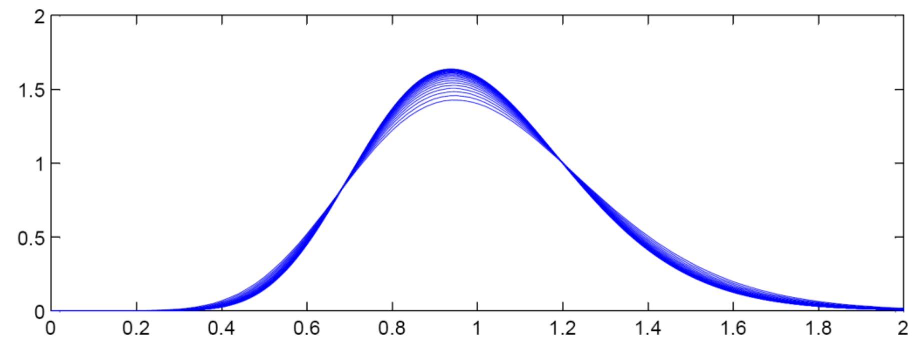
No parameter uncertainty: leads to unrealistic aggregate loss distribution as expected losses increase



If C is constant, NLR converges to 1.0 in distribution

Illustration shows aggregates with Poisson frequency and larger & larger values of E(A)

Including parameter preserves actual variability observed in data for large insurers



If C is not constant, NLR converges to C in distribution

Illustration shows aggregates with negative binomial frequency (gamma mixing) & larger & larger values of E(A)

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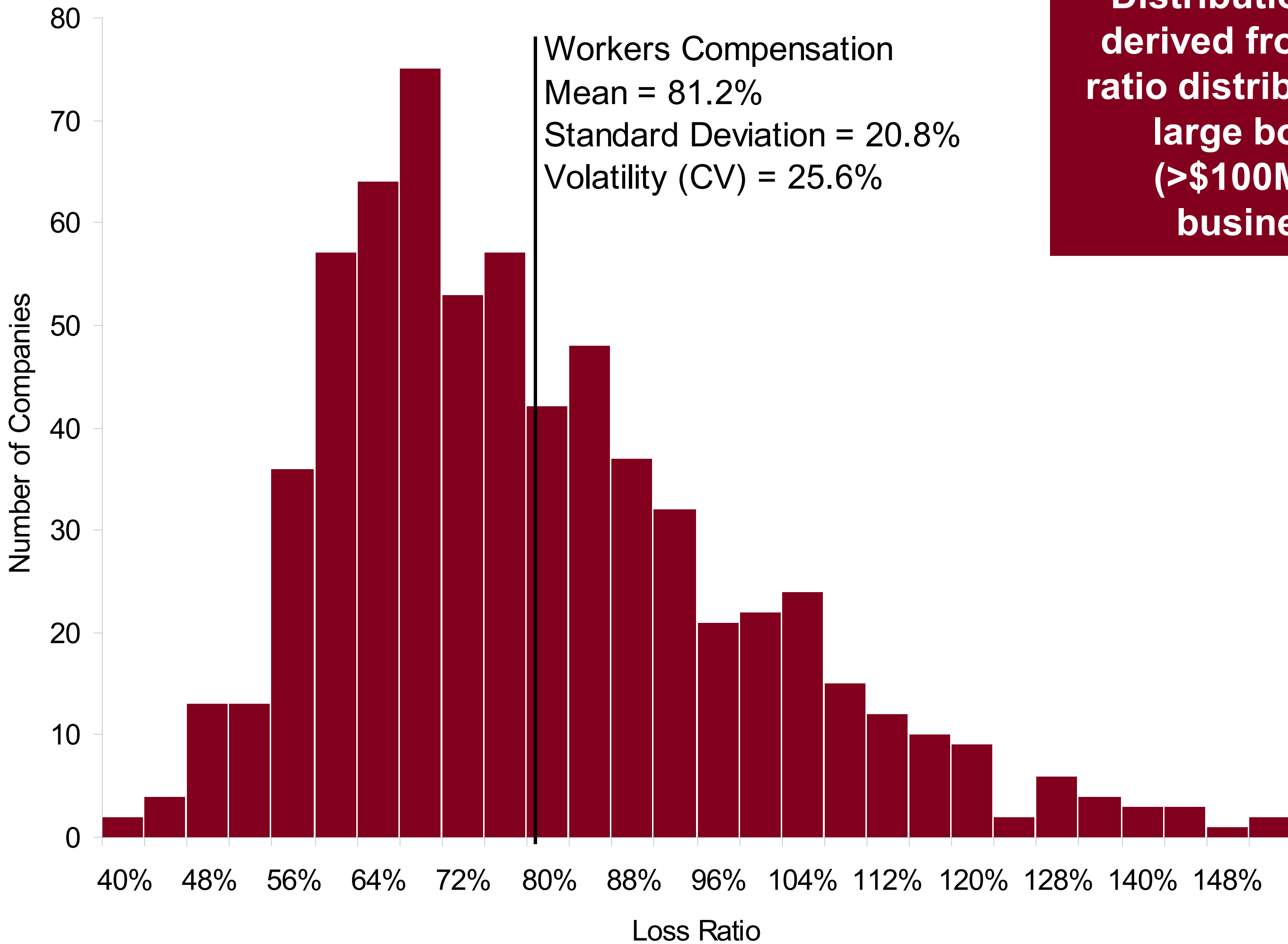
5. Empirical Evidence: Systemic Insurance Risk by Line



Distribution of C derived from loss ratio distribution of large books (>$100M) of business

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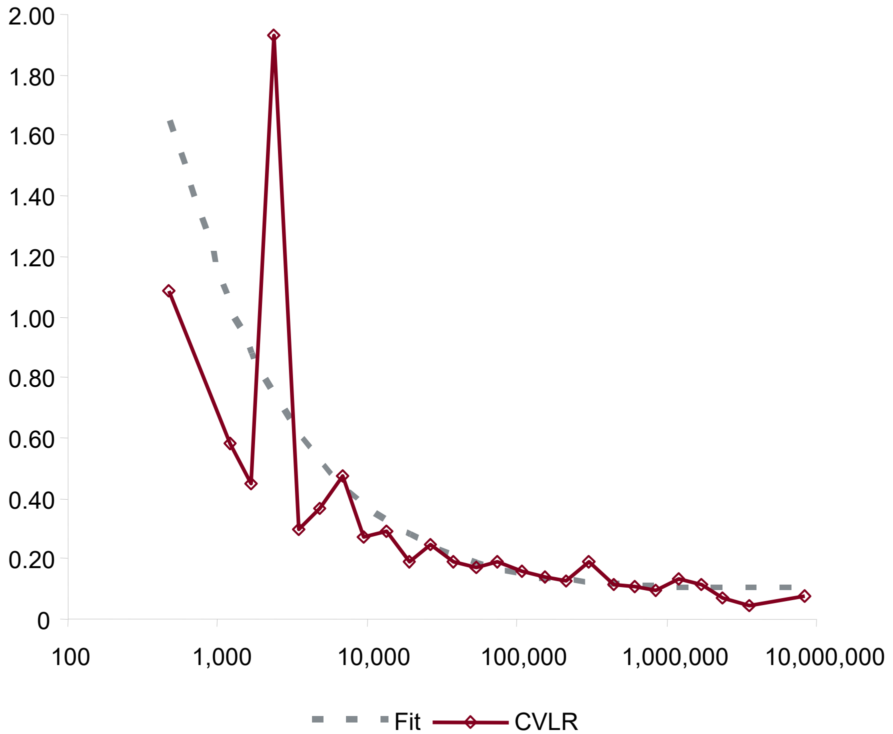
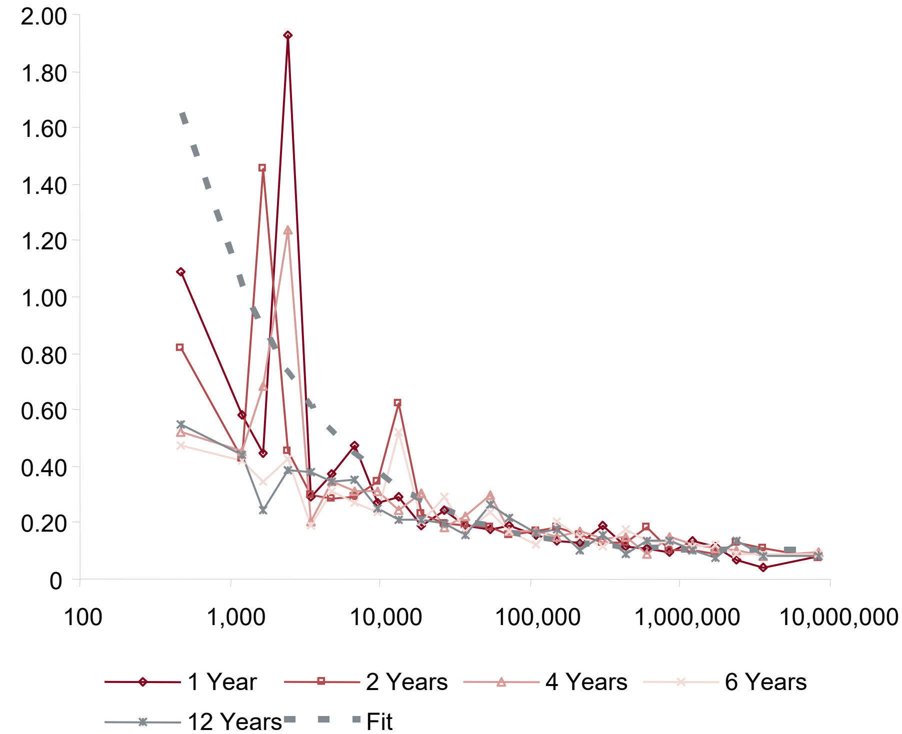
5. Empirical Evidence: Volumetric/Temporal Symmetry





- Consider volatility of A(x,t), A(2x,t/2), A(4x,t/4) etc.
- Same relationship between volatility and volume, xt
- **Data consistent with volumetric/temporal symmetry and with model A(x,t) = X(xCt)**

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6. Four Levy Process Models

- A(x,t) = X(xt) no Volumetrically diversifying
- A(x,t) = X(xZ(t)) no Volumetric/temporal asymmetry
- **A(x,t) = X(xCt) Yes Not volumetrically diversifying, volumetric/temporal symmetry**
- A(x,t) = X(xCZ(t)) no Volumetric/temporal asymmetry
- A(x,t) = xR(t) no Constant volatility with volume

Model	Variance	$v(x,t)$	Diversifying $x \to \infty$	Diversifying $t \to \infty$
$X(xt)$	$\sigma^2 xt$	$\frac{\sigma}{\sqrt{xt}}$	Yes	Yes
$X(xZ(t))$	$xt(\sigma^2 + x\tau^2)$	$\sqrt{\frac{\sigma^2}{xt} + \frac{\tau^2}{t}}$	No	Yes
$X(xCt)$	$xt(\sigma^2 + cxt)$	$\sqrt{\frac{\sigma^2}{xt} + c}$	No	No
$X(xCZ(t))$	$x^2t^2\left(\frac{(c+1)\tau^2}{t} + c\right) + \sigma^2 xt$	$\sqrt{\frac{\sigma^2}{xt} + \frac{\tau'^2}{t} + c}$	No	No
$xX(t)$	$x^2\sigma^2 t$	$\sigma/\sqrt{t}$	Const.	Yes

Variance and coefficient of variation v of each model

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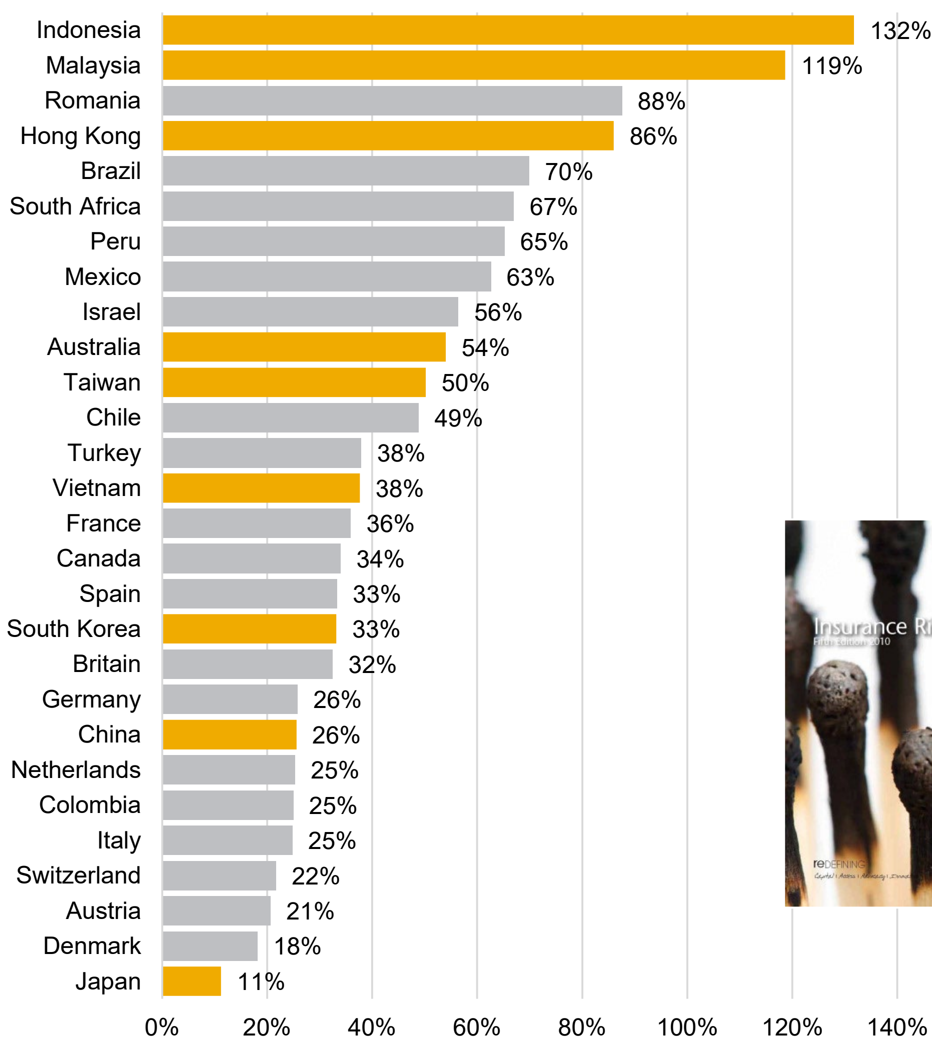
7. Application to APAC Region Countries: General Liability



- Fit coefficient of variation by country for General Liability
- Fit using CV = √(c + a / volume)
- Reporting asymptote √c
- Latest available statutory data
- Details published in Aon Benfield Insurance Risk Study, see http://thoughtleadership.aonbenfield.com/Pages/Home.aspx?reportcategory=Insurance Risk Study

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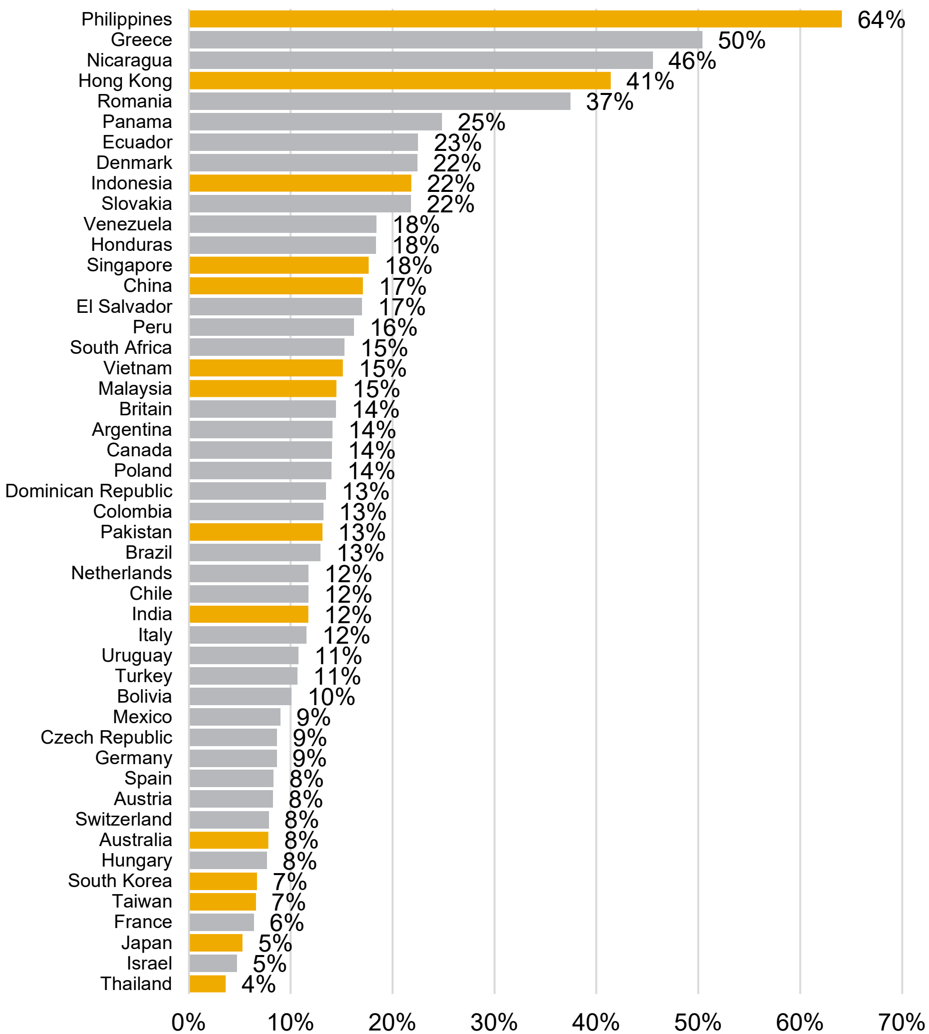
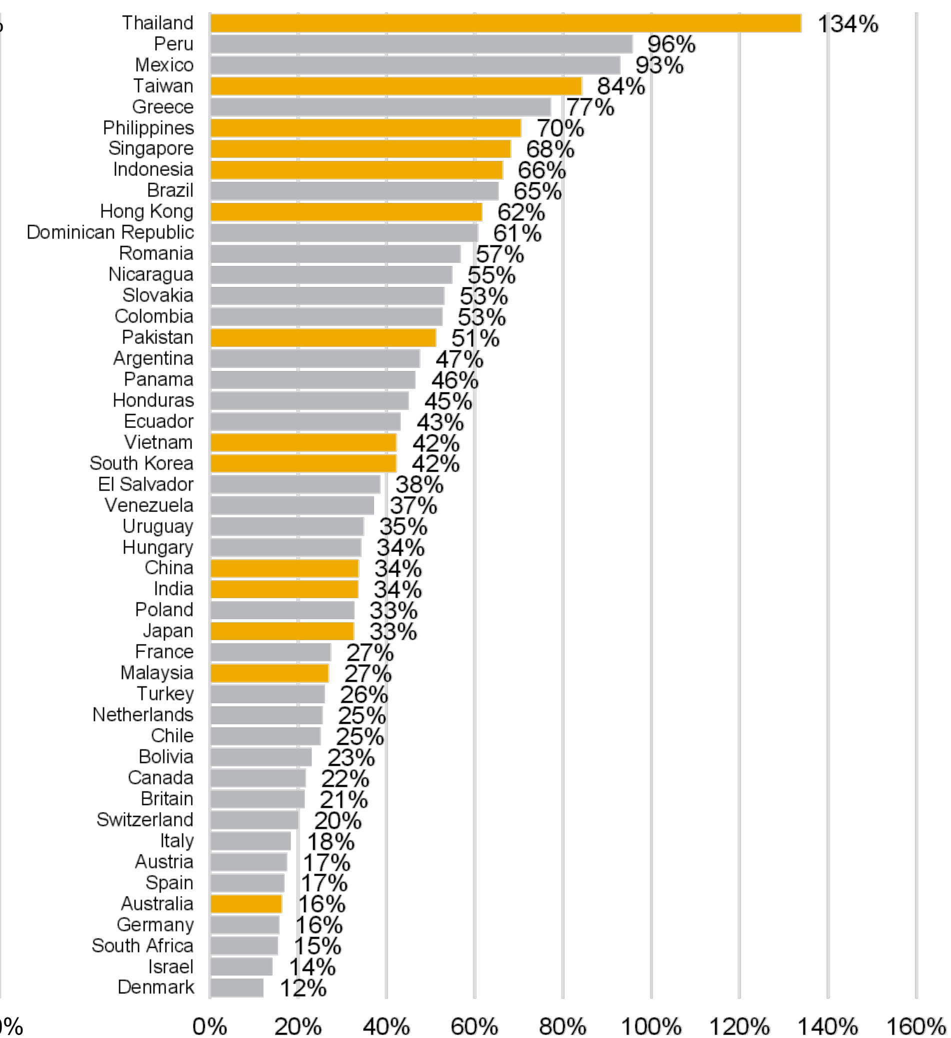
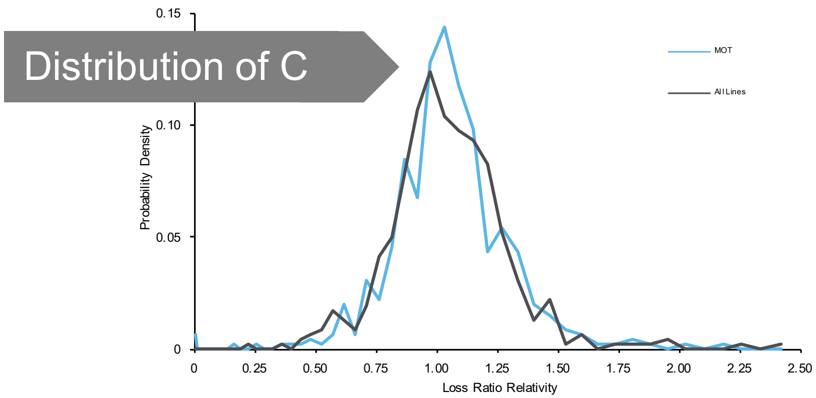
7. Application to APAC Region Countries: Motor (left) and Property

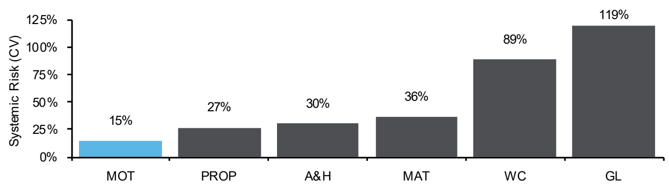






COEFFICIENT OF VARIATION	
MOT	14%
All Lines	20%

CBOE VIX	
Last	13
52wk Lo	11
52wk Hi	27



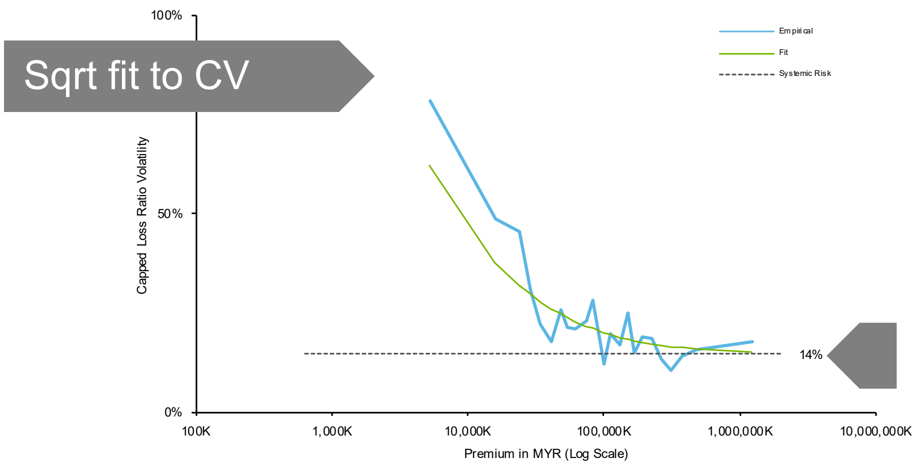
Currency	MYR	**Report Type**	Financial Year
Number of Companies	76	**Loss Ratios**	Case-incurred
Number of Observations	460	**LAE Type**	Loss Only

Loss Ratio Statistics	**All Observations**	**Largest 25**	**Largest 10**
Premium Threshold	0.0 M	501.3 M	1,130.4 M
Average Premium Size	183.5 M	1,097.3 M	1,711.8 M
Average Loss Ratio	71.3%	75.3%	73.9%
Std. Dev. Loss Ratio	19.3%	13.2%	17.4%
Loss Ratio Volatility (CV)	27.1%	17.6%	23.5%

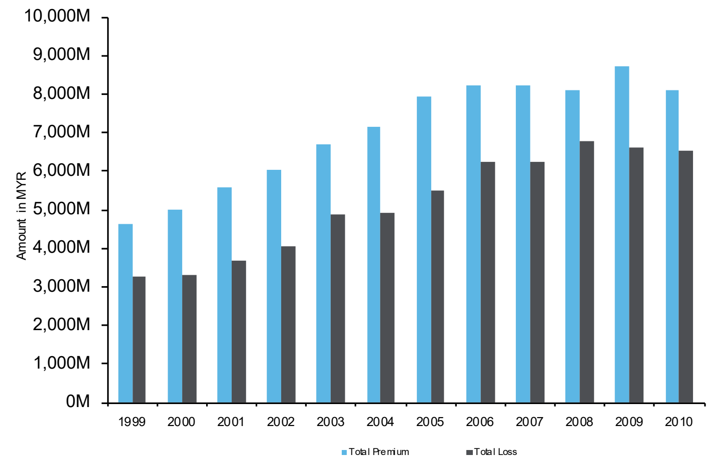
Loss Ratio Correlation Matrix

	A&H	GL	MAT	MOT	PROP	WC
A&H	100.0%	5.4%	14.7%	10.9%	8.4%	17.1%
GL	5.4%	100.0%	-2.5%	-7.7%	-1.5%	3.0%
MAT	14.7%	-2.5%	100.0%	-9.6%	7.5%	11.3%
MOT	10.9%	-7.7%	-9.6%	100.0%	-3.7%	5.0%
PROP	8.4%	-1.5%	7.5%	-3.7%	100.0%	4.9%
WC	17.1%	3.0%	11.3%	5.0%	4.9%	100.0%

By line correlation



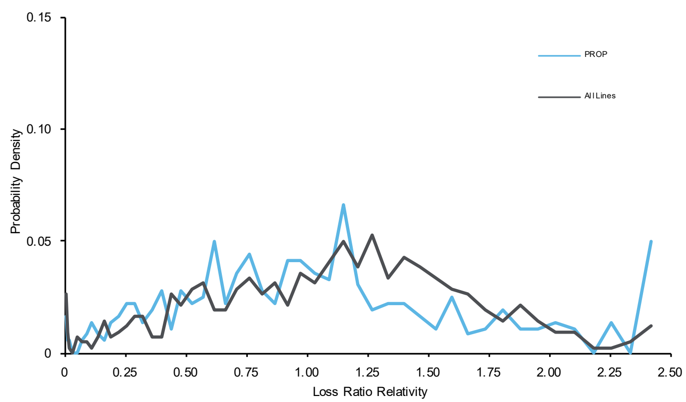
Gross Industry Premium and Loss, 1999 - 2010



Sources: Annual Insurance Statistics, Bank Negara Malaysia ; analysis by Aon Benfield Analytics; VIX prices as of Aug 13, 2013

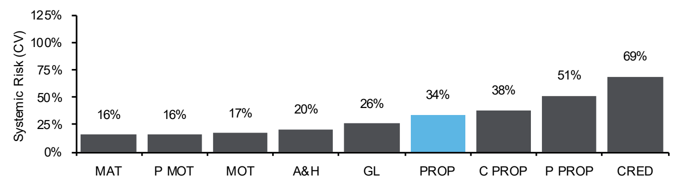
Property

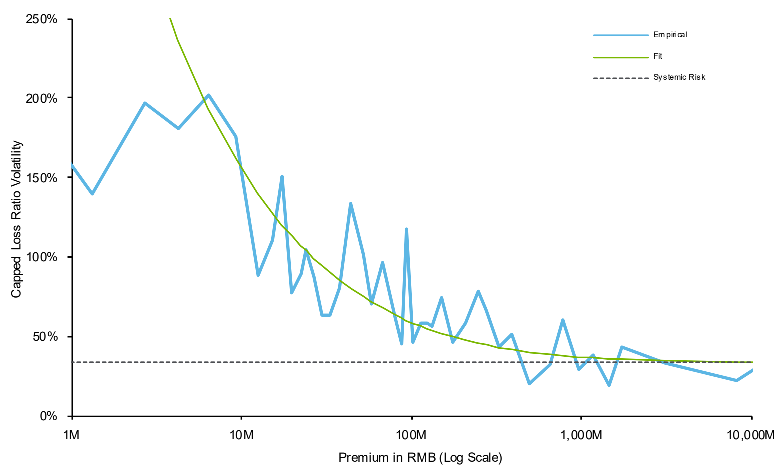
Gross Results for Financial Years 1997 - 2011



COEFFICIENT OF VARIATION	
PROP	34%
All Lines	17%

CBOE VIX	
Last	13
52wk Lo	11
52wk Hi	27





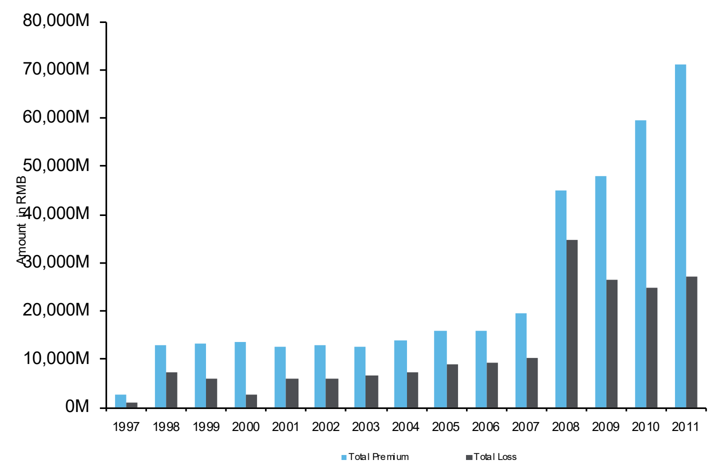
Currency	RMB	**Report Type**	Financial Year
Number of Companies	51	**Loss Ratios**	Paid
Number of Observations	364	**LAE Type**	Loss & ALAE

Loss Ratio Statistics	All Observations	Largest 25	Largest 10
Premium Threshold	10.0 M	2,381.6 M	9,708.9 M
Average Premium Size	1,008.8 M	10,178.0 M	15,165.0 M
Average Loss Ratio	34.6%	53.6%	50.0%
Std. Dev. Loss Ratio	27.7%	16.9%	20.0%
Loss Ratio Volatility (CV)	80.2%	31.5%	40.0%

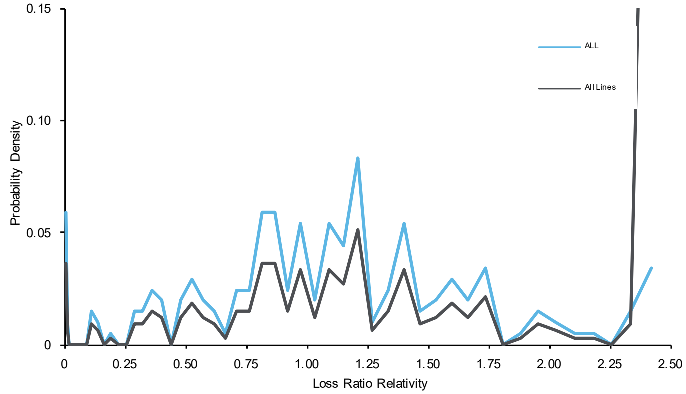
Loss Ratio Correlation Matrix

	A&H	CRED	GL	MAT	MOT	PROP
A&H	100.0%	18.4%	19.8%	18.7%	32.5%	55.1%
CRED	18.4%	100.0%	11.2%	15.2%	27.2%	21.0%
GL	19.8%	11.2%	100.0%	25.3%	25.7%	30.8%
MAT	18.7%	15.2%	25.3%	100.0%	31.5%	8.5%
MOT	32.5%	27.2%	25.7%	31.5%	100.0%	37.2%
PROP	55.1%	21.0%	30.8%	8.5%	37.2%	100.0%

Gross Industry Premium and Loss, 1997 - 2011



Sources: China Insurance Yearbooks; analysis by Aon Benfield Analytics; VIX prices as of Aug 13, 2013



COEFFICIENT OF VARIATION

ALL	18%
All Lines	18%

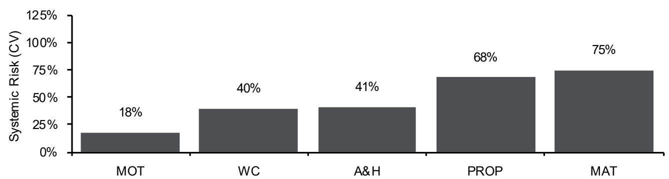
CBOE VIX	
Last	13
52wk Lo	11
52wk Hi	27

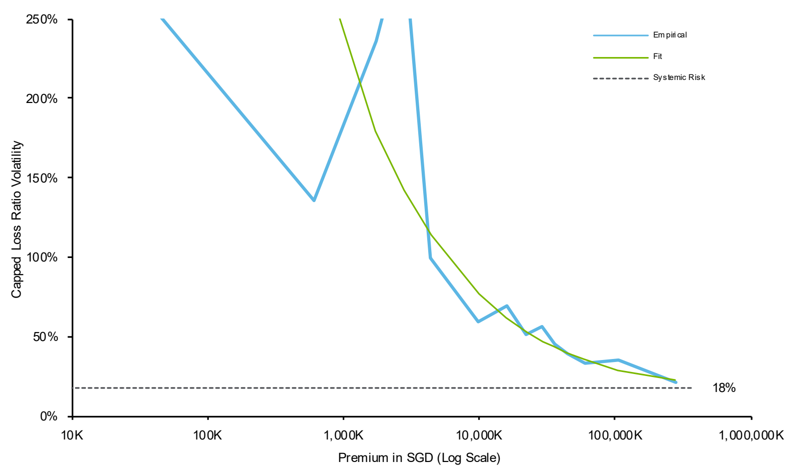
Currency	SGD	**Report Type**	Financial Year
Number of Companies	66	**Loss Ratios**	Paid
Number of Observations	204	**LAE Type**	Loss Only

Loss Ratio Statistics	**All Observations**	**Largest 25**	**Largest 10**
Premium Threshold	1.0 M	104.5 M	276.9 M
Average Premium Size	50.3 M	228.4 M	319.1 M
Average Loss Ratio	48.0%	57.7%	62.5%
Std. Dev. Loss Ratio	36.6%	16.2%	11.1%
Loss Ratio Volatility (CV)	76.3%	28.0%	17.8%

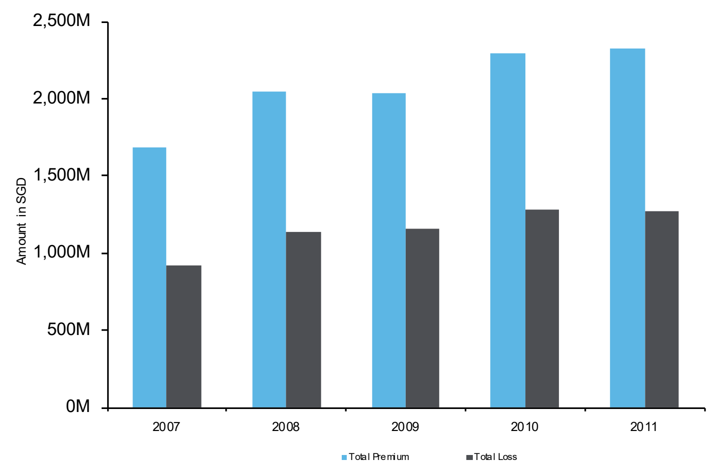
Loss Ratio Correlation Matrix

	A&H	MAT	MOT	PROP	WC
A&H	100.0%	-1.2%	15.0%	-7.1%	0.3%
MAT	-1.2%	100.0%	15.8%	9.2%	9.2%
MOT	15.0%	15.8%	100.0%	4.0%	35.5%
PROP	-7.1%	9.2%	4.0%	100.0%	3.1%
WC	0.3%	9.2%	35.5%	3.1%	100.0%





Gross Industry Premium and Loss, 2007 - 2011



Sources: Monetary Authority of Singapore - Insurance Report Archive; analysis by Aon Benfield Analytics; VIX prices as of Aug 13, 2013

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